The Current Developing Status on Grey System Theory*

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Abstract —— The scientific background against which the grey system theory has come into being, the astonishing progress that grey system theory has made in the world of learning and its wide application in many scientific areas is presented in this paper. The main reasons for the continued advance in grey system theory studies were revealed by the trends of system science and uncertainty system theory needs of social practice, and by principle of producing effects immediately from scientific theory, scientific method and scientific model, etc. This paper has certain reference value for founding or developing a new branch of learning.

Keywords: Grey system theory; Advances of science; Needs of society; Principle of producing effects immediately.

Sustaining Development of Grey System Theory

In 1982, Chinese Prof. Julong Deng's first grey system paper "The Control Problem of Grey System" was pioneered in "System & Control Letters" (Deng Julong, 1982(a)), published by North Holand publishing company. At the same year, his first Chinese grey system paper "The Grey Control System" was published in Journal of Huazhong

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University of Technology (Deng ulong, 1982(b)). Publication of these two creative papers indicated the birth of grey system theory as a new transverse subject, under fruitful efforts of its initiator Deng Julong during several years. As soon as birth of this new theory, it received positive attention from domestic and foreign academic circles and many practical workers, and many renowned scholars and experts affirmed and supported it vigorously. Many Chinese youth scholars joined in the group of grey system theory research, and made theory exploration and application research in different domains in passion. Especially, successful application of grey system theory in several scientific domains won affirmation and attention of international academic circles. At present, many well-known scholars have engaged in theory research and application of grey system in many countries, regions and international organizations like England, US, Germany, Japan, Australia, Canada, Austria, Russia, Taiwan, Hong Kong, United Nations and so on. The international English version of academic publication "The Journal of Grey System", originated in England in 1989, has become core periodical of important international digest organizations such as "Science Abstracts" (SA), "Mathematical Reviews" (MR), and so on. More than 1000 kinds of academic periodicals accept and publish grey system paper. The Official Journal of AC (Association for Computer), Fuzzy Mathematics Letters of Taiwan, and Cybernetes (SCI source periodical), the Official Journal of the World Organization of Systems and Cybernetics, have published special issues on grey system.

Some of universities have grey system theory curriculums and lectures, including Huazhong University of Science and Technology, Renmin University of China, Tsinghua University, Zhejiang University, Shandong University, Nanjing University of Aeronautics and Astronautics, American Maryland University, Toyohashi University of Technology, Kanagawa University, Vienna Economical University, French Astronavigation Center, Taiwan Central University, Chenggong University, Datong Engineering Institute, Tahsi University, Jianguo Industrial University, and son on. Huazhong University of Science and Technology, Nanjing University of Aeronautics and Astronautics, Fuzhou University, Wuhan University of Science and Technology, Southeast University in China and many universities in Taiwan raise doctors in grey system theory research. Thousands of doctors and masters utilize the ideas and methods of grey system theory to develop scientific research and write dissertations.

More than 100 academic works of grey system have been published by some domestic and foreign publishing organizations like Science Press, Defense Industry Press, Press of Huazhong University of Science and Technology, Science and

Technology Press of Jangsu, People's Press of Shangdong, References of Science Technology Press, Quanhua Science and Technology Books Press of Taiwan, Gaoli Books Limited Company of Taiwan, Technology Press of Japan, American IIGSS Academic Press, German Springer-Verlag Publish Company, and son on. A group of boundary subjects have been emerging, such as grey hydrology, grey geology, grey plant thremmatology, grey regional economy system analysis, grey philosophy. National, provincial and municipally science foundation sponsor grey system research positively in china, many research projects about grey system theory and application obtain kinds of subsidization. Statistics have indicated that there are more than 20 research findings of grey system obtaining reward from national or provincial authorities. In 2002, a grey system scholar from China won prize of World Organization on Systems and Cybernetics.

Many important international conferences arranged special sessions on grey system theory, such as International Conference on Uncertainty System Modeling (United States, March 1990), International Institute for General Systems Studies (Austin, United States, January 1997), the 12th International Congress of World Organization of Systems and Cybernetics (WOSC), the 4th Annual Meeting of International Institute for General Systems Studies (Pittsburgh, March 2002), the 32nd Computers & Industrial Engineering International Conferences (Limerick, Ireland. August 2003), IEEE International Conference on Systems, Men and Cybernetics (The Hague, The Netherlands, October 2004), IEEE International Conference on Network, Sensing and Control (Tucson, Arizona, United States, March 2005), the 13th International congress of World Organization of Systems and Cybernetics (WOSC, Malibor, Slovenia, July 2005), IEEE International Conference on Systems, Men and Cybernetics (Hawaii, USA, October 2005), IEEE International Conference on Systems, Men and Cybernetics (Taipei, October 2006), and so on. Grey System theory has become a hot topic in many important international conferences, which plays a positive role to make global system science circle understand grey system theory further more undoubtedly.

From 1982 to June in 2006, about 15,000 grey system papers were retrieved from Chinese academic periodical database in China National Knowledge Infrastructure (CNKI). Along them, 7309 papers were retrieved by subject "grey system". According to Table 1, the number of papers grows continually from 1982 to 2005, it is above 500 every year after 2002, and it achieves 270 from Jan to June in 2006.

Table 1. Retrieval results from Chinese academic periodical database.

Year	1982~2006		1982	1983	1984	1985	1986	1987	1988
Number	7309		6	4	6	11	42	82	125
Year	1989	1990	1991	1992	1993	1994	1995	1996	1997
Number	126	149	181	195	203	517	477	781	183
Year	1998	1999	2000	2001	2002	2003	2004	2005	2006
Number	448	456	418	435	512	556	550	576	270

From the retrieval results by imputing "grey system" in English electron resources EI Village database, the number altogether is 5986 from 1982 to June in 2006. We found that some of papers earlier don't belong to "grey system" discussed by this article, and above 95% percent of papers retrieved belongs to the "grey systems" category discussed by this article. We haven't reject papers not belonging category in this article in Table 2, but this does not affect our discussion. From Table 2, the number of grey system papers in EI Village database presents stable growing tendency.

Table 2. Retrieval results from EI Village database..

Year	1982~2006		1990	1991	1992	1993	1994	1995	1996
Number	5986		171	162	144	230	235	266	289
Year	1997	1998	1999	2000	2001	2002	2003	2004	2005
Number	333	291	266	326	359	400	399	574	615

During the later half of 20th century, in the field of Systems Science and Engineering, a variety of systems theory and methodology had been emerging constantly. Some new theories were paid little attention after initial time and most of new theories were concerned to different extents, but many new theories are lack of powers to impulse its continuous growth. Like any new subject, grey system theory attained positive attention and support from academic circle at earlier times. At the same time, it was inevitable to be criticized and doubted. Facing various kinds of criticism and doubt, some people who joined in the group of grey system theory research turned to other research areas for worrying about their work that couldn't be accepted by the public, and some people stop research work because of the age problem after retirement. But, some people who persist in going on, and fresh people who took part in the research continually promoted sustaining growth of grey system theory.

Important Reasons for Sustaining Growth of Grey System Theory

Grey system theory conforms to the trend of system science and uncertainty system theory

Based on widespread divisions in activities of scientific research, the highly synthetic tendency has brought forward many cross disciplinary research activities possessing significant methodological meanings. The systems science has revealed more profoundly and essentially some important internal relations among the subjects, which have deeply promoted the integrative progress of modern science and technology. With the help of these newly emerging fields of study, many complicated problems, unsolvable before, can be resolved successfully and more deeper understandings about the nature have been brought forward (Gu, J. and Wu, G., 1999). These Cross disciplinary theories include, to say a few, system theory, information theory and cybernetics, which were formulated during the end of the 1940s, theory of dissipative structures, synergetics and fractals, which started to be known during the end of the 1960s and the beginning of 1970s, ultracircular theory and general systems theory, which have been more maturing after late 1970s.

During the systems research, because of the disturbance from both inside and outside and the limitation of cognitive level, the information people get is always uncertain. With the development of science and technology and the progress of social society, people's understanding about the uncertainties of various systems is much more profound than ever before, and the studying on it is also more in-depth. During the later half of 20th century, in the field of Systems Science and Engineering, a variety of systems theory and methodology on uncertainty had been emerging constantly. For instance, Prof. L.A. Zaden had established fuzzy mathematics in the 1960s, Professor Deng Julong had pioneered a difficult and fruitful research on grey system theory, Professor Z. Pawlark had initiated Rough Set Theory in the 1980s and Professor Wang Guang-yuan had contributed a great deal in unascertained mathematics. All those above are significant achievements in researches on unascertained systems, and they also expound the theories and methodologies on describing and dealing with numerous unascertained information from different aspects.

Grey system theory is a new method for studying uncertain problem with less data and poor information. The new theory studies on the "small sample", "poor

information" systems with "partial information known, partial information unknown". It describes correctly and monitors effectively system's operation and evolution, through extracting valuable information from known information. Grey system theory has come into being with development of system sciences group and uncertainty system theory and methods, and conforms to current system science and uncertain system theory. It is also the result of deepening perceptivity to uncertain system.

Grey system theory conforms to urgent needs of social practice
Grey system theory was established at the beginning of the 1980s. At that time, China came into the new period of reform and opening up, gong with the central task of economic construction, and development turned into most important task to all sectors and regions. To guide social, economic and technological healthy development, the government needs to formulate scientific plans for middle or long-term development. As we all know, planning must be based on the in-depth analysis of historical data and scientific prediction of future development prospect. According to scientific diagnosis of real system's operation, to define system's advantages and constraints, understand developing rule of things, then we can compile scientific, perspective, feasible program planning. Traditional system diagnosis, analysis, prediction, decision-making models usually require a large number of data. But China's statistical system then resumed shortly, collectable data are very limited. Grey system theory as a new theory addressing systems with "partial information known, partial information unknown" and "small sample" as well as "poor information" can be applied to assessment, diagnosis, analysis, modeling, forecast, control, optimization of "small sample" and "poor information" uncertainty system, which conforms to the urgent needs of Chinese social practice.

Since the 1980s, grey system theory, models and techniques have been used in hundreds of cities, counties and provincial-level regions of China to work out their strategic planning of unified development, and promote healthy development of regional society, economy and technology. Grey system theory as a unique, applicable, effective method of soft science, in completely spontaneous circumstances, was widely applied, resulting in tremendous impacts. In 1993, a "Blue Book of Science and Technology of China (No. 8)" was compiled and published by China Science and Technology Commission, in which grey system theory was affirmed as a new soft science method established by Chinese scholars, (China Science and Technology Commission, 1993). In our daily social economic and scientific research activities, we

often face situations of incomplete information. For example, in some studies of agriculture, even though all the information related to the planted area, the quality of seeds, fertilizers, irrigation, etc., is completely known, it is still difficult to estimate the production quantity and the consequent annual income due to various unknown or vague information related to labor quality, technology level employed, natural environment, weather conditions, etc. As for the case of insects control, we might have known very well the relation between the special kind of insect and its natural enemies, but it might still be difficult for us to achieve the desirable effects due to the reason that we do not have enough information regarding relations between the insect of our concern and the baits, its natural enemies and the baits, one natural enemy and other natural enemies, one kind of insect and other kinds of insects, etc. For each adjustment of a price system in our economy, decision makers often face the difficulty of not knowing the definite information on the effect of the price change on consumers, on the prices of goods, etc. All liquid pressure systems are difficult to control due to some immeasurable quantities. Electricity systems are hard to observe because of the stochastic parameters of the voltage and currents, which is caused by not having enough knowledge on motion and parameters. In a general social or economic system, it is difficult to analyze the effect of the input on the output for the reasons that there do not exist clear differences between the "interior" and the "exterior", the system self and its environment, and that the boundary of the system may be sometime easy to tell or on other occasions difficult to clarify. In scholastic works, a same economic variable could be seen as endogenous by some scholars and external by some other scholars. The appearance of such phenomenon is due to the lack of modeling information, or the reason that an appropriate systems model has not been found, or the fact that the right observation and control variables have not been employed.

The application scope of Grey System Theory has extended to industry, agriculture, social affairs, economy, energy, transportation, oil, geology, water conservancy, meteorology, ecology, environment, medicine, education, sports, military, legal, financial and other fields, and have resolved a large number of practical problems in production, life and scientific research successfully. For example, trap grey forecast of Tarim Basin in Xinjiang completed by Professor Zhongxiang Wu; grey forecast of the oil reserves distribution of the west in South China Sea completed by Tech Changjung Lin; grey forecast of the Yellow River estuary terrain evolution completed by Tech Zhaowu Fan; grey clustering analysis of Henan Oilfield terrane completed by Professor Pan Heping; destroying obstacles scheme in the landing program selection completed

by Weiguo Shi and others; the grey forecast of the mud-rock flow changing trends completed by Zhongxin Jiang and others; grey diagnostic systems on traditional Chinese-Western medicine developed by Researcher Xuejun Qiu — all the above produced notable social and economic benefits.

Essential Reasons for Continual Growth of Grey System Theory

Grey system theory conforms to the principle of producing immediately the effects of scientific theory

In scientific development history, simplicity, practicality, and efficiency (producing effects immediately), have been nearly believed by all the scientists. Back in the sixth century BC, natural philosophers understood the material world in a common desire: reducing the material world to some simple common elements for purposes of research. Ancient Greek mathematician and philosopher Pythagoras (Pythagoras) brought forward four elements (soil, water, fire, gas) theory that the material is constituted by the four elements in about 500 BC. There are also five elements theory in ancient China that the basic of everything is allied to five things, namely, metal, wood, water, fire and soil.

Newton's law of mechanics unified macroeconomic phenomena as a simple form. In "Natural Philosophies Principle of Mathematics", Newton said: "The nature don't do useless thing, just doing a little is all right, instead doing more is useless; as nature likes simplification, but doesn't love boasting itself with other extra reasons." In the era of relativity, Einstein proposed two standards of the test theory: "certified external" and "internal completeness" that is "logical simplicity." He said that, from the view that scientific theories reflect the harmony and order of the nature, real scientific theory would be simple, practical, effective (producing effects immediately) principle (Zhiqiang Hu, 2005, Liangying Xu, 1976).

In 1970s, Amperes, Weber, Lai vine, Grassmannian, Maxwelland so on have a theory to explain the electromagnetic phenomena form different assumptions. As Maxwell's theory most accords with Principle of producing effects immediately, so his theory was the most widely circulated (Weiping Sun, 1999).

Based on the thorough pondering for social, economic system forecast, control problems, professor Deng Julong have advanced grey system theory creatively due to

he discovered a kind of uncertainty system which with "partial information known, partial information unknown" and "small sample", "poor information" that can't be described by existing uncertainty system theory such as probability statistics and fuzzy mathematics, etc. (Liu Sifeng, 1996, 2004). The new theory, taking the grey number as the basic unit, from the explanation of "the grey system" concept, based on "difference information principle", "solution non-unique principle", "the least information principle" and other basic principles. It has established the analysis system which depends on the grey relational space; the method system which based on the grey sequence operator and the sequence generation; the model system which takes grey model (GM) as the core; the technical system which takes system appraisal, diagnosis, analysis, modeling, forecast, decision-making, control. Optimizing as it's main body, all of them conform to the principle of producing effects immediately of scientific theory.

It is believed by principle of producing effects immediately of scientific theory that the world is a uniform and harmonious whole, governed by very simple principle due to a small amount of logic form. Upon establishing such theories, we can grasp the development laws of things. Grey system theory considers social system, economic system, the technological system, the ecological system etc. as the generalized energy system, and according to the reality that energy accumulation and release generally consistent with the exponential distribution, it brought forward a series of GM model with exponential distribution (Deng Julong, 1990), which can be widely applied to the actual system.

Grey system methods conform to principle of producing the immediately effects of scientific methods

Descartes (R. Descartes), ancestor of modern deductive pointed out that "a problem can be divided into each small part until it can be satisfactorily resolved". This saying explains function of the whole simplified into part best. By using the principle of producing effects immediately of scientific methods, it can simplify irregular rules to regular, uneven to uniform, anisotropic to isotropic, linear to nonlinear, smooth to not smooth, limited to unlimited (or in reverse), continuous to discontinuous (or in reverse), and high-dimensional to low-dimensional, isolation system to non-isolation system and son on so that it can explore ways and means to resolve the problem (Xuete Yang, 2005).

Domination theory in synergetic is a typical example of this aspect, through

eliminating faster relaxation variables of high-dimensional nonlinear equations of describing the evolution of system according to dominant theory, we can put the original high-dimensional equations into low-dimensional order parameter evolution. Since the order parameter determines the dynamics in the vicinity of a critical point, by solving the order parameter evolution equation, we can obtain time structure, space structure or time-space structure of a system. So that it can control operation of the system effectively (Haken, H. 1978).

In the economic area, both methods for using Gini coefficient to describe income gap and for using Cobb-Douglas production function to measure the share of contributions of technical improvement in economic growth are based on the simplification of actual system. They are all based on the principle of producing effects immediately of scientific methods.

Grey relational analysis, clustering evaluation, sequence operator and grey sequence generation, grey cluster modeling theory and some of representative methods and techniques such as forecast, decision-making controlling and optimization in grey system, all established for the sake of simplifications of real system or through the simplifications of real system, and they conforms to the principle of producing effects immediately of scientific methods.

The grey system modeling conforms to the principle of producing immediately the effects of the scientific modeling

Scientific model, the core of scientific method, is the key to solving practical problems with the application of scientific knowledge. The conformation of the scientific model must comply with the principle of producing effects immediately.

The logical structure of the model is simpler and easier than prototype, easy to perform logic ratiocination, mathematical illation and experimental operation. Principle of producing effects immediately of the scientific model is achieved mainly by depending on the simplification and development of the model representations and the deletion secondary factors of the system. In the history of scientific development, broadly spread scientific models are those that comply with principle of producing effects immediately.

For example, based on a lot of observational data on the planets, Kepler discovered the famous Kepler's Third Law of Planetary Motion: $T^2=D^3$. Namely the square of planetary revolution cycle is equivalent to the cube of its distance form the sun. It is a very simple form. So is the Modigliani's Model:

$$\frac{C_t}{y_t} = a + b\frac{y_0}{y_t}, \quad a > 0, \ b > 0$$

which is applied to describing the average propensity to consume (APC) and Phillips' curve

$$\frac{\Delta p}{p} = a + b\frac{1}{x}$$

which is used to describe the relationship between the inflation rate $\dfrac{\Delta p}{p}$ and

unemployment x. It is also true for the famous capital asset pricing model (CAPM):

$$E[r_i] = r_f + \beta_i\left(E[r_m] - r_f\right)$$

All of them can transform into the simplest simple linear regression models in fact.

The basic form of grey model GM(1,1) model

$$x^{(0)}(k) + az^{(1)}(k) = b$$

applied mostly is also a simple linear regression model.

The shadow equation

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b$$

of GM(1,1) model is a first-order ordinary linear differential equations (Liu Sifeng, 2004; Liu Sifeng and Yi Lin, 1998, 2005).

Based on differential principles, when the value of the development coefficient a lies in a certain range, the difference equation can be approximately replaced by the differential equation. Then, the GM(1,1) response time sequence

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}, \quad k = 1, 2, ..., n.$$

is obtained, which can be used to forecast.

Grey system model from the model thoughts to the model forms happens to hold the same view with the principle of producing effects immediately of the scientific model.

Concluding Remarks

More than twenty years have passed since grey system theory was pioneered by Prof. Deng Julong in 1982. The young people who join in theoretical research of grey system in the 1980's have been graying at two temples now. In the long history of scientific development, the 20 years pass in a flash. Many scientific theories require the unremitting efforts of several generations of people and have gone through hundreds of years before reaching maturity and perfect. Grey system theory is over 20 years old, it is still in the growth period, and it is unavoidable that there exist immature and imperfect parts in grey system theory. People engaged in grey system theoretical research should welcome and take serious in all criticisms. And then problems and the flaws can be overcame unceasingly, the new growing point be excavated unceasingly, exploring unceasingly, innovating unceasingly, thus making the grey system theory, originated by Chinese scholars, go forward unceasingly.

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On Brand Effect

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Introduction

The existing energy from actions of brand effect contributes to the progress of human society. Thus brand effect refers to intellectual energy. How to utilize brand effect in making much more contribution to human society progress is a vital objective to be achieved.

Definition of brand effect field: Let $U(p)$ be an intellectual information, we thus say it is a brand effect field provided that

(1) $U(p)$ comprises the sole key unit, as an issuing order unit, along with some followers, as receiving order unit;
(2) $U(p)$ is closed

Definition of brand effect level: The distance from the marketing point to warehouse is said to be brand effect level (BEL), provided that there is a rich revenue obtained via the marketing.